

# Tactics and Metaprogramming

An introduction to Meta-F<sup>\*</sup>

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**F<sup>\*</sup>** PoP Up Seminar

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# This Talk

- High-level presentation + some live examples at the end.
- No way we cover everything today.
- See new chapter (in construction) in **F\*** tutorial.

# Main uses of Meta-F<sup>★</sup>

- Simplifying/proving assertions: `assert  $\varphi$  by  $\tau$`
- Metaprogramming, in various flavors.
- Solving implicit arguments (typeclasses).
- Program transformations (e.g. for efficient extraction).

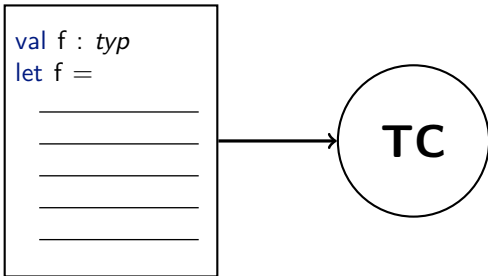
# Proving

# Auto-active Verification

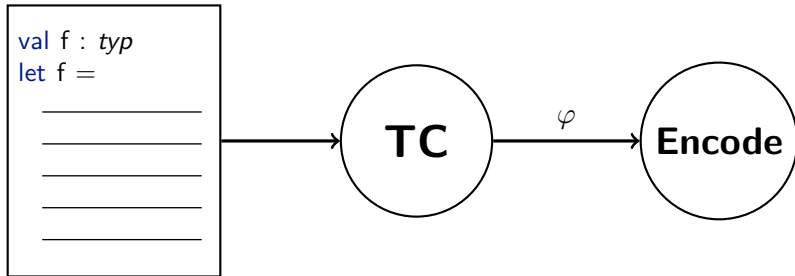
```
val f : typ  
let f =
```

```
_____  
_____  
_____  
_____  
_____
```

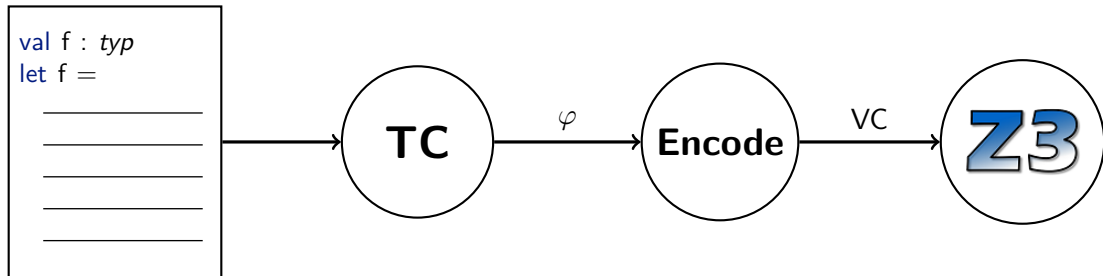
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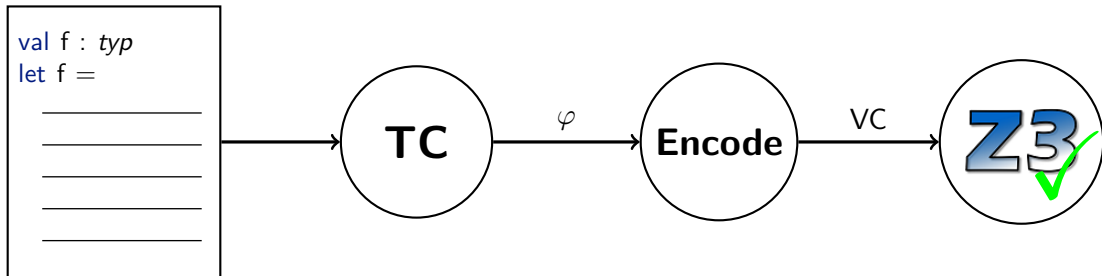


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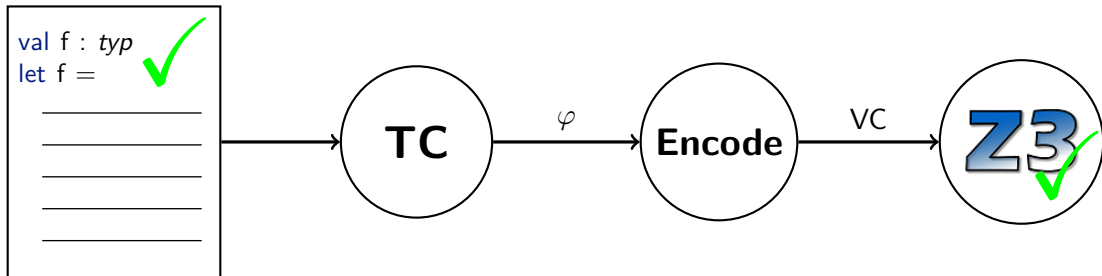




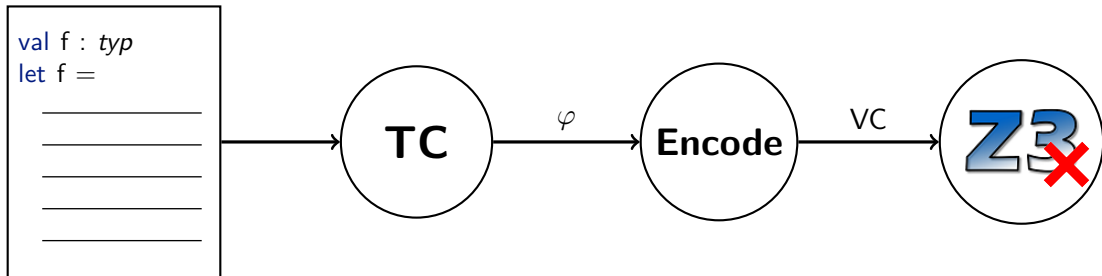
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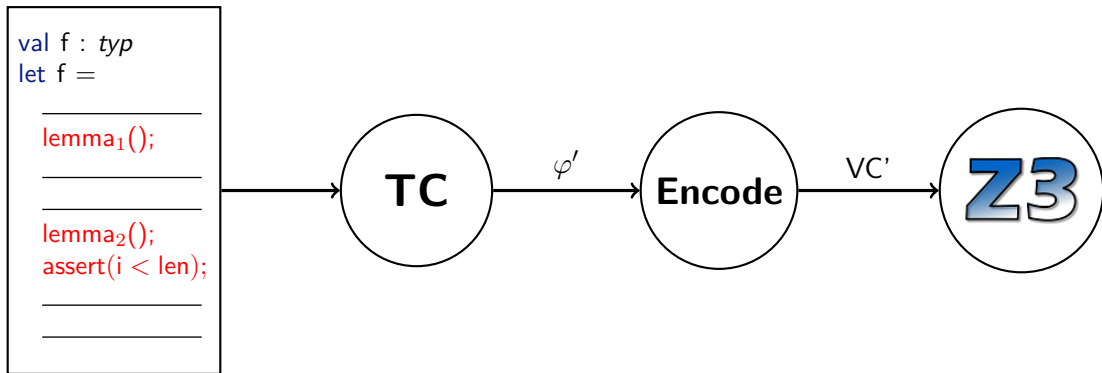
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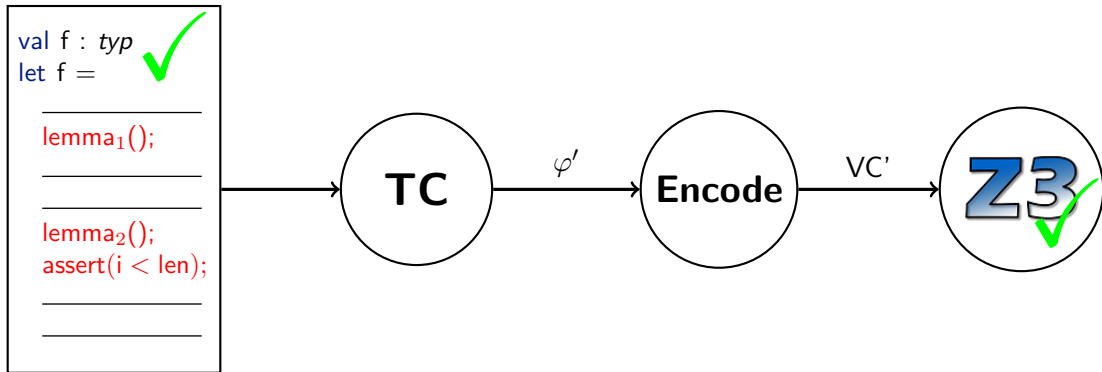
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# Proofs by SMT

Z3 fleshes out many proofs easily. Normally, the “key insights” are all that is needed.

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val mod_add_both (a:int) (b:int) (x:int) (n:pos) : Lemma
  (requires a % n == b % n)
  (ensures (a + x) % n == (b + x) % n)
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let mod_add_both (a:int) (b:int) (x:int) (n:pos) = ()
```



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let mod_add_both (a:int) (b:int) (x:int) (n:pos) =
  lemma_div_mod a n;
  lemma_div_mod b n;
  lemma_div_mod (a + x) n;
  lemma_div_mod (b + x) n;
  let xx = (b + x) / n - (a + x) / n - b / n + a / n in
  distributivity_sub_left ((b + x) / n) ((a + x) / n) n;
  distributivity_sub_left ((b + x) / n - (a + x) / n) (b / n) n;
  distributivity_add_left ((b + x) / n - (a + x) / n - b / n) (a / n) n;
  lt_multiple_is_equal ((a + x) % n) ((b + x) % n) xx n
```

# When SMT *really* doesn't cut it

```
let lemma_poly_multiply (n p r h r0 r1 h0 h1 h2 s1 d0 d1 d2 hh : int)
  : Lemma
    (requires  $p > 0 \wedge r1 \geq 0 \wedge n > 0 \wedge 4 * (n * n) == p + 5 \wedge r == r1 * n + r0 \wedge$ 
       $h == h2 * (n * n) + h1 * n + h0 \wedge s1 == r1 + (r1 / 4) \wedge r1 \% 4 == 0 \wedge$ 
       $d0 == h0 * r0 + h1 * s1 \wedge d1 == h0 * r1 + h1 * r0 + h2 * s1 \wedge$ 
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    (ensures  $(h * r) \% p == hh \% p$ )
  =
let r1_4 = r1 / 4 in
let h_r_expand = (h2 * (n * n) + h1 * n + h0) * ((r1_4 * 4) * n + r0) in
let hh_expand = (h2 * r0) * (n * n) + (h0 * (r1_4 * 4) + h1 * r0 + h2 * (5 * r1_4)) * n
               + (h0 * r0 + h1 * (5 * r1_4)) in
let b = ((h2 * n + h1) * r1_4) in
modulo_addition_lemma hh_expand p b;
assert (h_r_expand == hh_expand + b * (n * n * 4 + (- 5)))
```

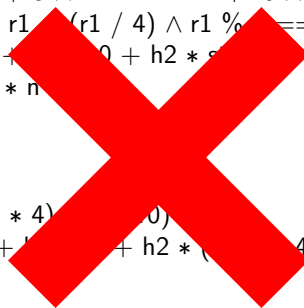
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Rather, pre-processing the VC before SMT.

- Arithmetic canonicalization (nlarith)
- Simplification via rewrites, normalization
- Case splitting
- Proof environment manipulation (e.g. fact pruning)



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- Arithmetic canonicalization (nlarith)
- Simplification via rewrites, normalization
- Case splitting
- Proof environment manipulation (e.g. fact pruning)
- Fully automating proofs also possible

# Tackling individual assertions

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let b = ((h2 * n + h1) * r1_4) in
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assert (h_r_expand == hh_expand + b * (n * n * 4 + (- 5))) by (canon_semiring int_cr)
```

# Splitting assertions

VC will not contain the obligation,  
instead we get a *goal* for it

$\forall n \text{ p r } \dots,$

$\varphi_1 \implies \psi_1 \wedge$

$\varphi_2 \implies \psi_2 \wedge$

$\dots \implies L = R \wedge$

$L = R \implies \dots$

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$$\begin{aligned} \forall n \ p \ r \ \dots, \\ \varphi_1 \implies \psi_1 \wedge \\ \quad \varphi_2 \implies \psi_2 \wedge \\ \quad \quad \dots \implies \textcolor{red}{L} = \textcolor{red}{R} \wedge \\ \quad \quad \quad L = R \implies \dots \end{aligned}$$

Goal 1/1

$n : \text{int}$

$p : \text{int}$

$r : \text{int}$

...

$H0 : \varphi_1$

$H1 : \varphi_2$

...

---

$L = R$

- Can switch between part of a VC (formula) and a goal (type-theoretic hole)

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```
type error = exn * proofstate (* error and proofstate at the time of failure *)  
let tac a = proofstate  $\rightarrow$  Dv ((a & proofstate) 'either' error) (* Dv: possibly diverging *)  
let t_return (x: $\alpha$ ) = ...  
let t_bind (m:tac  $\alpha$ ) (f: $\alpha \rightarrow$  tac  $\beta$ ) : tac  $\beta$  = ...
```

```
new_effect { TAC with repr = tac ; return = t_return ; bind = t_bind }
```

```
sub_effect DIV  $\rightsquigarrow$  TAC = ...
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- No put operation, can only modify proofstate via primitives:  
exact, apply, intro, tc, raise, catch, ...

# TAC is a first-class citizen of $F^*$

Metaprograms are written as any other kind of program:

```
let mytac () : Tac unit =  
  let h1 = implies_intro () in  
  rewrite h1;  
  apply_lemma ('mylem);  
  ...
```

# TAC is a first-class citizen of $F^*$

Higher-order combinators too:

```
let rec repeat (t : unit → Tac  $\alpha$ ) : Tac (list  $\alpha$ ) =  
  match catch t with  
  | Inl _ → []  
  | Inr x → x :: repeat t
```

```
let repeat1 (t : unit → Tac  $\alpha$ ) : Tac (list  $\alpha$ ) = t () :: repeat t
```

# TAC is a first-class citizen of $F^*$

Use exceptions:

```
exception NotAForall
```

```
let forall_intro () : Tac binder =  
  let g = cur_goal () in  
  match term_as_formula g with  
  | Forall _ _ → begin apply_lemma ('fa_intro_lem); intro () end  
  | _ → raise NotAForall
```

```
let t () =  
  try forall_intro () with  
  | NotAForall → ()  
  | e → raise e
```

# TAC is a first-class citizen of $F^*$

Call into existing pure, diverging, and exception-raising code:

```
let f (seen:list term) : Tac unit =  
  let g = cur_goal () in  
  if FStar.List.Tot.Base.existsb (term_eq g) seen then  
    raise Loop;  
  ...
```

# Metaprogramming



# Metaprogramming: generating terms

Beyond proving, Meta-**F**<sup>\*</sup> enables constructing terms and top-level definitions

```
let one = _ by (exact ('1))  
let one_plus_two = _ by (apply ('(+)); exact ('1'); exact ('2'))
```

# Metaprogramming: generating terms

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```
let one = _ by (exact ('1))
let one_plus_two = _ by (apply ('(+)); exact ('1); exact ('2))
let fib (i:nat) = if i < 2 then exact ('1)
                  else begin
                    apply ('(+));
                    fib (i - 1);
                    fib (i - 2)
                  end
let t = _ by (fib 8)
```

# Metaprogramming: generating terms

Beyond proving, Meta-**F**<sup>\*</sup> enables constructing terms and top-level definitions

```
noeq type t1 =  
  | A : int → string → t1  
  | B : t1 → int → t1  
  | C : t1  
  | D : string → t1  
  | E : t1 → t1  
  
%splice (mk_printer ('t1))
```

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```
%splice (mk_printer ('t1))
```

```
let t1_print = λv →  
  let rec ff_rec v_inner =  
    (match v_inner with  
     | A a1 a2 →  
       concat "" [ "(";  
                   concat " " ["A"; print_int a1; print_string a2];  
                   ")" ]  
     | B a1 a2 → concat ""  
                 [ "("; concat " " ["B"; ff_rec a1; print_int a2]; ")" ]  
     | C → "C"  
     | ...  
    in ff_rec v
```

# Customizing implicit arguments

Meta-**F**<sup>\*</sup> can also be used to provide strategies for resolution of implicits.

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let diag (x:int) (#[same_as x] y : int) : int * int = (x, y)
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let diag (x:int) (#[same_as x] y : int) : int * int = (x, y)
```

```
let _ = assert (diag 42 == (42, 42))
```

```
let _ = assert (diag 42 #21 == (42, 21))
```

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module FStar.Typeclasses
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module MyMod
noeq type deq a = {
  eq : a → a → bool;
  eq_ok : (x:a) → (y:a) → Lemma (eq x y  $\iff$  x == y)
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(* val eq : #a:Type → ([tcresolve] d : deq a) → (a → a → bool) *)
(* val eq_ok : #a:Type → ([tcresolve] d : deq a) → ((x:a) → (y:a) → Lemma ...) *)
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(* val eq : #a:Type → [| deq a |] → (a → a → bool) *)
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[@tcinstance] let eq_int : deq int = { eq = ( $\lambda$  x y → x = y); eq_ok = easy }
```

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module MyMod
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  eq : a → a → bool;
  eq_ok : (x:a) → (y:a) → Lemma (eq x y  $\iff$  x == y)
}
(* val eq : #a:Type → [| deq a |] → (a → a → bool) *)
(* val eq_ok : #a:Type → [| deq a |] → ((x:a) → (y:a) → Lemma ...) *)

instance eq_int : deq int = { eq = ( $\lambda$  x y → x = y); eq_ok = easy }
```



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- This works, but is very slow due to the constant use of the typechecker/unifier.

# Proof-by-reflection: 1 minute pitch

- Obtain an AST of the expression.

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- Stay tuned for chapter in **F**\* tutorial, this week.
- Also check out `examples/` and `tests/`

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Demo!